# Relativistic Processes and the Internal Structure of Neutron Stars

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**Abstract.**
Models for the internal composition of Dense Compact Stars are reviewed as well as macroscopic properties derived by observations of relativistic processes. Modeling of pure neutron matter Neutron Stars is presented and crust properties are studied by means of a two fluid model.

**Keywords:** neutron star, nuclear matter, equation of state, phase transition
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## INTRODUCTION

Neutron stars (NS) are stellar objects that represent one possible end of stellar evolution. They are in hydrostatic equilibrium in which the gravitational force is balanced by the internal one resulting from the Pauli exclusion principle. Theorized first by Landau (1932) and Zwicky (1934), Neutron Stars have been first observed as pulsars in 1967 by Jocellyn Bell. The latest observations of NS point out that the most extreme physical conditions are present in them. They are the densest compact objects ($\rho \sim 10^{14} - 10^{16}$ g$/$cm$^3$), fastest rotating stars (as fast as 716 MHz as measured) and fastest moving objects in the galaxy ($v \sim 1083$ km$/$s). They possess the highest magnetic fields ($B = 10^{15}$ G and largest surface gravity ($10^{14}$ cm$/$s$^2$). Their interior contains superconducting material with the highest expected temperature value ($T_c = 10^9$ K), and even neutrinos can be trapped in proto-neutron stars with temperatures at birth of about 700,000 million K [1].

## NEUTRON STARS MODELS

Given such extreme conditions various theoretical models have been proposed to describe the composition of dense nuclear matter which are translated into an Equation of State (EoS). Based on our present knowledge of nuclear interactions, EoSs can be categorized in a general form as

- (Hadronic) Pure neutron matter: mostly n, but also p, e and $\mu$ particles.
- (Hadronic) Hyperonic matter: n, p, hyperons($\Lambda$, $\Sigma$, $\Xi$) and leptons (e, $\mu$).
- (SQM) Strange Quark Matter. Deconfined quarks, where the name goes to the heaviest quark that appears as density increases (in this case the strange quark).
- Boson Condensates ($\pi$, K): Bosons can be present as a Bose-Einstein condensate state of matter (BEC).

In terms of these EoSs dense compact stars can be classified as:

- Neutron Stars: both crust and core are described by a hadronic EoS.
- Hybrid Stars: their crust is hadronic but having a quark matter core.
- Quark Stars: Only described by a Quark Matter EoS.

The macroscopic, relativistic structure of a Neutron Star is described as follows. For a static, non-rotating star, the Einstein equations give the Tolman-Oppenheimer-Volkoff equations [2]:

$$\frac{dp}{dr} = -\frac{(\rho + p/c^2)G(m + 4\pi r^3 p/c^2)}{r^2(1 - 2Gm/rc^2)}, \qquad \frac{dm}{dr} = 4\pi r^2 \rho. \tag{1}$$

This system requires an EoS of the form $p(\rho)$ and is to be solved for mass $m$, pressure $p$ and density $\rho$ inside the star. To find a solution it's necessary to choose a central density ($\rho_c$) and take into account the boundary conditions that $m(r=0)=0$, $m(R)=M$ and $p(r=R)=0$ where $R$ and $M$ are the total radius and total mass of the star. As for the crust, the moment of inertia $I$ and mass $M$ are derived, in the frame of General Relativity [3], as:

$$I \simeq \frac{J}{1 + 2GJ/R^3c^2}, \qquad J = \frac{8\pi}{3}\int_0^R r^4 \left(\rho + \frac{p}{c^2}\right), \tag{2}$$

$$\Delta I_{crust} = \frac{2}{3}(M_{crust}R^2)\frac{1 - 2GI/R^3c^2}{1 - 2GM/Rc^2} \tag{3}$$

where $M_{crust} = M - M_{core}$ the difference between the total mass and the mass of the core. To determine the latter the crust-core transition point must be known, i.e. the edge of the solid crust where the liquid core starts. The EoS of a neutron star is based on nuclear models that describe nuclear matter,i.e. a system of interacting nucleons. For the star to be stable, charge neutrality (total charge resulting from all its constituents) and beta equilibrium (beta and inverse beta reactions taking place at the same rate) must hold. Electrons (and at enough high densities muons) form a gas that is negatively charged. The crust of the star forms a lattice of nuclear clusters immersed in neutron liquid and as such has solid state properties. The core behaves like a liquid and presents a mixture of Fermi gases of protons, neutrons and leptons. In the core, the nuclear energy per baryon is a function of only baryon number density $n$ and the isospin asymmetry $\alpha$ and is defined as follows:

$$E_{nuc}(n,\alpha) = V(n) + S(n) * \alpha^2 + Q(n) * \alpha^4 + \mathcal{O}(\alpha^6) \tag{4}$$

where $\alpha \equiv (n_n - n_p)/n$ and $n \equiv n_p + n_n$ are the neutron and proton number densities. Here the last term is negligible since it's contribution is very small. The most interesting quantity is the Symmetry Energy (SE) $S(n)$ that has impact in the crust properties of the star. It's value at saturation point (the density of the nucleus of an atom), $n_0 = 0.16$, is $S(n_0) = 30 \pm 1$ MeV, as determined by experiments of isospin diffusion [4] and in agreement with the semi-empirical mass formula used to describe nuclei. Values at low density have lately been investigated both in theory and experiment [5, 6].

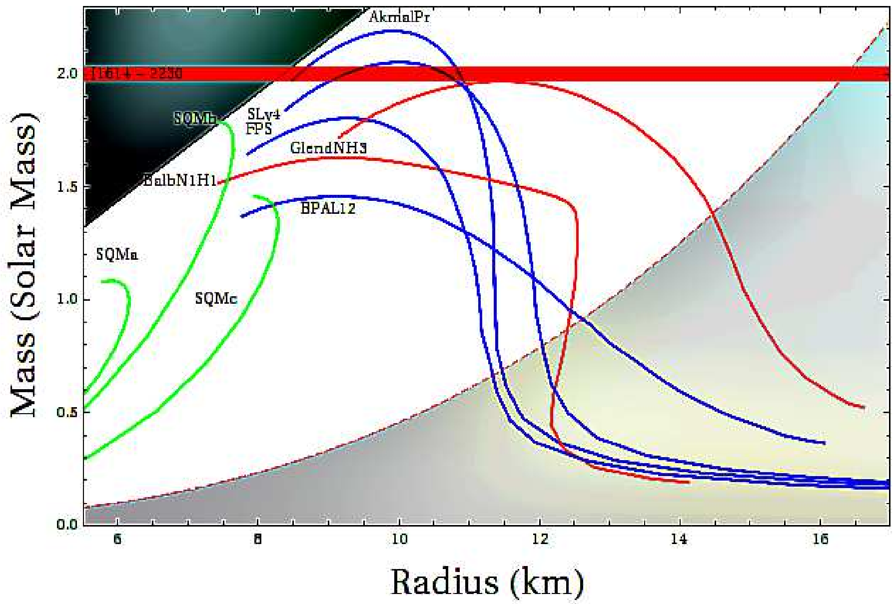


**FIGURE 1.** Mass vs Radius Relation for EoSs described in Table 1 for non-rotating NS. Blue lines represent hadronic EoSs for pure NS. Pink lines are hadronic EoSs that include hyperons. Green lines are SQM EoSs. The dashed orange line is the rotational limit imposed by the fastest rotating NS. The dark line in the left upper corner represents the causality limit that EoSs must respect. The red band at the 2 $textrmM_{\odot}$ is the measured value for the J1614-2230 NS. More details in [7] from where the figure is adapted.

The core system is then described by thermodynamic quantities derived from this energy form (for example $p = -\partial E/\partial V$) at zero Temperature ($T = 0$) since its contribution has no effects in the EoS. To estimate the crust core transition point one may start by looking for the instability values against density fluctuations where the system must split into two phases i.e. where the compressibility of nuclear matter becomes negative, since the condition $K_\mu = \left(\frac{\partial p}{\partial n}\right)_\mu \geq 0$ must hold. This marks a lower bound in density values where the transition should occur. The line corresponding $K_\mu = 0$ is usually called spinodal line. A more elaborated way of addressing this problem is to consider a first order phase transition of a two component system. The first consisting only of neutrons while the second having neutrons and protons. To ensure stability, mechanical and chemical equilibrium must take place by means of the Gibbs conditions:

$$p^I = p^{II} \quad , \quad \mu_n^I = \mu_n^{II} \quad , \quad \mu_e^I = \mu_e^{II}. \tag{5}$$

Protons are present only in second phase, so for them $\mu_p^I > \mu_p^{II}$. In the two component system the average densities and energies are described in terms of the volume fraction $\chi$ occupied by the "I" fluid. They are defined as:

$$\langle \varepsilon \rangle = \chi \varepsilon^I + (1-\chi)\varepsilon^{II} \quad , \quad \langle n \rangle = \chi n^I + (1-\chi) n^{II}. \tag{6}$$

This energy as a function of density is to be compared with the energy of the homogeneous system, the transition point being the value for which the two component system

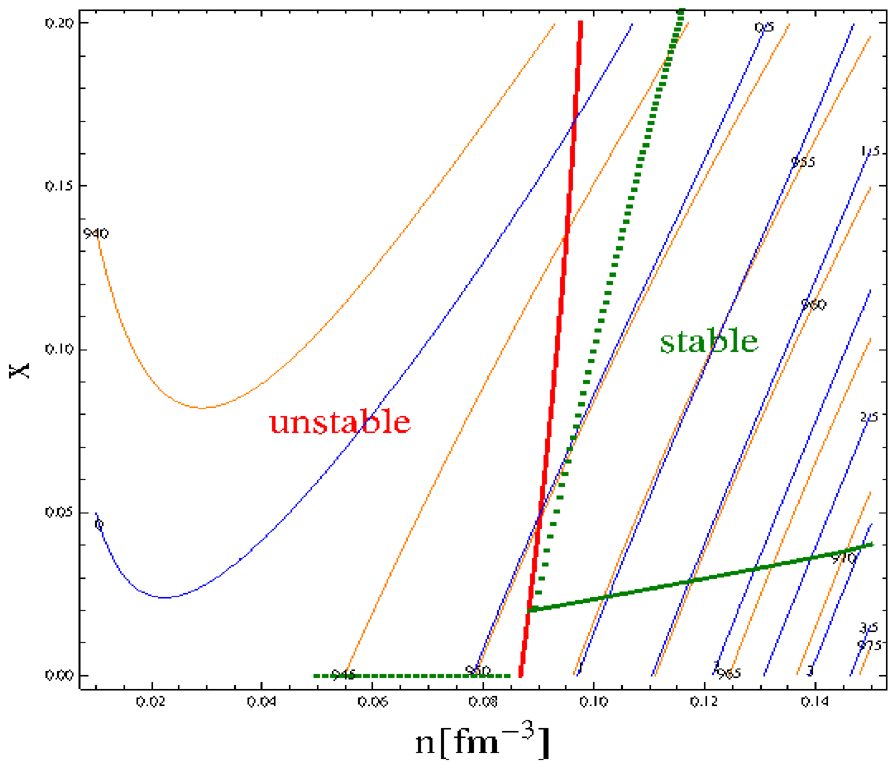


**FIGURE 2.** Diagram for the two component system for the PALu [8]. Blue contours represent nucleonic pressure while orange ones neutron chemical potential. The spinodal line is in red. Green dots are the points where the Gibbs conditions are fulfilled. Points to the left of the spinodal belong to the pure neutron phase, on the right are points for the nuclear clusters. The crust-core transition takes place at the density where the green solid line touches the spinodal. For this model it occurs at $n_c = 0.0892$ fm$^{-3}$.

is preferred, where its energy is the lowest. The behaviour of matter close to the crust-core transition region is shown in Fig.2. Any state of matter is represented by a point on the $x - n$ plane, where $x$ is the proton fraction $x \equiv n_p/n$ and $n$ is the baryon number density. The green solid line corresponds to the homogeneous neutral matter filling the NS core. As the density decreases the homogeneous matter becomes unstable when it crosses spinodal line (the red one). All states of matter on left side of the spinodal are not stable except the pure neutron matter ($x = 0$). That means that for low density the system splits into two phases: nuclear matter $x \neq 0$ in the form of finite size clusters immersed in pure neutron matter (almost free neutron + electron gas). Those two phases are represented by two branches of green points for which the coexistence conditions (5) hold. In this approach the finite size effect like Coulomb and surface energy are neglected. These different effects lead to formation of various structures like rods, plates, and are called pasta phases. See [9] for a detailed discussion.

## RELATIVISTIC MEASUREMENTS

Astronomical observations can shed light on NS macroscopic properties and help to rule out theoretical models. The two basic quantities are mass $M$ and radius $R$. Fig.1 shows

the expected values for families of stars given an EoS. Most processes like Eddington flux near the Star's surface, redshift of any luminous signal and spectra from thermal bursts involve the compactness parameter $\frac{GM}{Rc^2}$ where only the ratio of these quantities is found [10, 11]. Nevertheless, combining at least two processes is possible to find such values, something that has not yet been achieved. For this task, future observations with new extraterrestrial telescopes that have been implemented are quite promising. In the case of orbiting binary starts only $M$ can be derived with good accuracy in the case of orbit reduction (by gravitational wave emission) and Shapiro delay (radar signal delay near massive objects) lately measured and resulting in a 2 $M_{\odot}$ NS [12]. Macroscopic crust properties of NS have also been constrained by pulsar glitching: a sudden spin up of the star. For the case of the Vela pulsar a lower bound on the crustal moment of Inertia, $I/I_{crust} \geq 1.4\%$, has been established by observations based on the superfluity model [13].

## SUMMARY

Neutron Stars are dense compact objects where the most extreme physical conditions exist. The macroscopic description being in terms of the General Theory of Relativity. For the interior of the star different nuclear models are being used based on terrestrial laboratory experiments and extrapolated to the conditions found in NS. Theoretical models should be able to reproduce observations, and with the forthcoming data such models should start to converge since some of them could be tweaked in their parameters to reach the desired values. In particular the latest 2 $M_{\odot}$ measurement imposes a strong constrain in the EoS. Finally other relativistic processes like the cooling of the proto-neutron star could also provide evidence on the interior of NS, since the cooling rate is modified by the presence of superfluid matter. All this is reflected in the $M$ vs $R$ relation.

**TABLE 1.** EoSs used in Fig.1

| Symbol | References and Specifications |
|---|---|
| AkmalPR | Akmal et al.1998 A18+dv+UIX* (npemu) core, BPS+HP94 outer crust, SLy4 inner crust. |
| SLy4 | SLy4 (npemu) core, BPS+HP94 outer crust, SLy4 inner crust, Douchin and Haensel 2001. |
| FPS | BPS below n.drip, then FPS. |
| BPAL12 | Prakash 1997. |
| BalbN1H1 | BalbN1H1 core, SLy4 crust (S.Balberg,October 1997). |
| GlendNH3 | Nucleons + Hyperons, Lagrangian mean field theory, Glendenning, N.K. 1985 ApJ293, 470 . |
| SQMa | SQM EoS: $\varepsilon = p/v^2 + \varepsilon_b$, $\varepsilon_b = 5.6\varepsilon_0$, $v^2 = 1/3$, Glendenning, August 1990. |
| SQMb | SQM EoS: $\varepsilon = p/v^2 + \varepsilon_b$, $\varepsilon_b = 5.6\varepsilon_0$, $v^2 = 1$, Glendenning, August 1990. |
| SQMc | SQM EoS: $\varepsilon = p/v^2 + \varepsilon_b$, $\varepsilon_b = 3.1\varepsilon_0$, $v^2 = 1/3$, Glendenning, August 1990. |

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